$\boxed{P3}$ 30% of A Levels | DIFFICULTY ↑.
75 MARKS
1 h 50 min
11 Questions

Binomial
Partial
Polynomials
Logs
Trig
DIFF
integ
Diff eqr

P3 BINOMIAL

n is integer > 1, {n = 2, 3, 4, 5.....}

P1 BINOMIAL $(a+b)^n = a^n + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 \ldots$

P3 BINOMIAL

must be 1

when n is negative or fraction/decimal

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \ldots$$

CONDITION $-1 < x < 1$

FACTORIAL: $3! = 3 \times 2 \times 1$ $\boxed{x!}$

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$1! = 1$, $0! = 1$

Q: Expand first four terms in expansion of

(i) $(1+2x)^{-1}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!}$$

$$= 1 + (-1)(2x) + \frac{(-1)(-1-1)(2x)^2}{2!} + \frac{(-1)(-1-1)(-1-2)(2x)^3}{3!}$$

$\boxed{(-1)(-1-1)(2)^2 \div 2! = 4}$ $\boxed{(-1)(-1-1)(-1-2)(2)^3 \div 3! = -8}$

$$= 1 - 2x + 4x^2 - 8x^3$$

$\boxed{2}$ $(1-2x)^{\frac{1}{2}}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \ldots$$

$$1 + \left(\frac{1}{2}\right)(-2x) + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\frac{(-2x)^2}{2!} + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\frac{(-2x)^3}{3!}$$

$\boxed{(1\div 2)(1\div 2-1)(-2)^2 \div 2! = -\frac{1}{2}}$ $\boxed{(1\div 2)(1\div 2-1)(1\div 2-2)(-2)^3 \div 3! = -\frac{1}{2}}$

$$1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3$$

$\boxed{3}$ $\left(1-\frac{2}{3}x\right)^{\frac{3}{2}}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \ldots$$

$$= 1 + \left(\frac{3}{2}\right)\left(-\frac{2}{3}x\right) + \left(\frac{3}{2}\right)\left(\frac{3}{2}-1\right)\left(-\frac{2}{3}x\right)^2$$

$$(2)(3) \qquad \frac{(2)(2)(3)}{2!}$$

$$\boxed{(3 \div 2)(3 \div 2 - 1)(-2 \div 3)^2 \div 2! = \frac{1}{6}}$$

$$= 1 - x + \frac{1}{6}x^2$$

$$(1 + x)^{n} = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \dots$$

$\downarrow$

IF THIS IS NOT 1....

$(3x + 2)^{-1}$ Correct form: $(\text{constant} + \text{variable})^n$

$(2 + 3x)^{-1}$

$\cancel{2\left(1 + \frac{3}{2}x\right)^{-1}}$ $\longrightarrow$ MOST IMPORTANT BLUNDER TO REMEMBER.

$$\left[2\left(1 + \frac{3}{2}x\right)\right]^{-1}$$

$$2^{-1}\left(1 + \frac{3}{2}x\right)^{-1}$$

$$\frac{1}{2}\left[1 + (-1)\left(\frac{3}{2}x\right) + \frac{(-1)(-1-1)}{2!}\left(\frac{3}{2}x\right)^2\right]$$

$$\boxed{\frac{1}{2}\left(1 - \frac{3}{2}x + \frac{9}{4}x^2\right)}$$

$\boxed{2}$ $(x+2)^{-2}$ Expand first three terms.

$(2+x)^{-2}$

$\left[2\left(1+\frac{x}{2}\right)\right]^{-2}$

$2^{-2}\left(1+\frac{x}{2}\right)^{-2}$

$\frac{1}{4}\left[1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-2-1)}{2!}\left(\frac{x}{2}\right)^2\right]$

$\frac{1}{4}\left[1-x+\frac{3}{4}x^2\right]$

Q: $f(x) = -\frac{1}{x-1} + \frac{4}{x-2} - \frac{2}{x+1}$

Show that if expanded upto x^3 term,

$f(x) = -3 + 2x - \frac{3}{2}x^2 + \frac{11}{4}x^3$ (5 mark)

$\boxed{\text{Solution}}$ $f(x) = -\frac{1}{x-1} + \frac{4}{x-2} - \frac{2}{x+1}$

$-1\underbrace{(x-1)^{-1}}_{W1} + 4\underbrace{(x-2)^{-1}}_{W2} - 2\underbrace{(x+1)^{-1}}_{W3}$

$-1\left[-1(1+x+x^2+x^3)\right] + \overset{2}{\cancel{4}}\left[-\frac{1}{\cancel{2}}\left(1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}\right)\right] - 2\left[1-x+x^2-x^3\right]$

$$1 + x + x^2 + x^3 - 2 - x - \frac{x^2}{2} - \frac{x^3}{4} - 2 + 2x - 2x^2 + 2x^3$$

$$-3 + 2x - \frac{3}{2}x^2 + \frac{11}{4}x^3$$

(W1) $(x-1)^{-1}$

$(-1+x)^{-1}$

$[-1(1-x)]^{-1}$

$(-1)^{-1}(1-x)^{-1}$ $\boxed{(-1)(-1-1)(-1)^2 \div 2! = 1}$ $\boxed{(-1)(-1-1)(-1-2)(-1)^3 \div 3! = 1}$

$$-1\left[1+(-1)(-x)+\frac{(-1)(-1-1)(-x)^2}{2!}+\frac{(-1)(-1-1)(-1-2)(-x)^3}{3!}\right]$$

$\boxed{-1\left(1+x+x^2+x^3\right)} \longrightarrow$ (W1)

(W2) $(x-2)^{-1}$

$(-2+x)^{-1}$

$\left[-2\left(1-\frac{x}{2}\right)\right]^{-1}$

$(-2)^{-1}\left(1-\frac{x}{2}\right)^{-1}$

$$-\frac{1}{2}\left[1+(-1)\left(-\frac{x}{2}\right)+\frac{(-1)(-1-1)}{2!}\left(-\frac{x}{2}\right)^2+\frac{(-1)(-1-1)(-1-2)}{3!}\left(-\frac{x}{2}\right)^3\right]$$

$\boxed{\frac{-1}{2}\left(1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}\right)} \longrightarrow$ (W2) .

(W3) $(x+1)^{-1}$

$(1+x)^{-1}$

$$1+(-1)(x)+\frac{(-1)(-1-1)(x)^2}{2!}+\frac{(-1)(-1-1)(-1-2)(x)^3}{3!}$$

$\boxed{1-x+x^2-x^3} \longrightarrow$ (W3) .

29 Show that, for small values of x^2,

$$(1-2x^2)^{-2}-(1+6x^2)^{\frac{2}{3}} \approx kx^4,$$

where the value of the constant k is to be determined. [6]

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(W1) $(1-2x^2)^{-2} = 1 + (-2)(-2x^2) + \dfrac{(-2)(-2-1)(-2x^2)^2}{2!}$

$= 1 + 4x^2 + 12x^4$

(W2) $(1+6x^2)^{\frac{2}{3}} = 1 + \left(\frac{2}{3}\right)(6x^2) + \left(\frac{2}{3}\right)\left(\frac{2}{3}-1\right)\dfrac{(6x^2)^2}{2!}$

$= 1 + 4x^2 - 4x^4$

$(1-2x^2)^{-2} - (1+6x^2)^{\frac{2}{3}} \approx Kx^4$

$[1+4x^2+12x^4] - [1+4x^2-4x^4]$

$\cancel{1} + \cancel{4x^2} + 12x^4 \cancel{-1} - \cancel{4x^2} + 4x^4$

$16x^4 \approx Kx^4$

$\boxed{K = 16}$

6 **(i)** Simplify $\left(\sqrt{(1+x)}+\sqrt{(1-x)}\right)\left(\sqrt{(1+x)}-\sqrt{(1-x)}\right)$, showing your working, and deduce that

$$\frac{1}{\sqrt{(1+x)}+\sqrt{(1-x)}}=\frac{\sqrt{(1+x)}-\sqrt{(1-x)}}{2x}.$$ [2]

(ii) Using this result, or otherwise, obtain the expansion of

$$\frac{1}{\sqrt{(1+x)}+\sqrt{(1-x)}}$$

in ascending powers of x, up to and including the term in x^2. [4]

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Simplify $= \left(\sqrt{1+x}+\sqrt{1-x}\right)\left(\sqrt{1+x}-\sqrt{1-x}\right)$

$$\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2$$

$$(1+x)-(1-x)$$

$$1+x-1+x$$

$$2x$$

$$\frac{1}{\sqrt{1+x}+\sqrt{1-x}}=\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$$

cross multiply

$$2x = \left(\sqrt{1+x}+\sqrt{1-x}\right)\left(\sqrt{1+x}-\sqrt{1-x}\right)$$

from first part this is $2x$

$$2x = 2x.$$

$$\boxed{\frac{1}{\sqrt{1+x}+\sqrt{1-x}}} = \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$$

we can't apply Binomial on this.

Lets expand RHS.

(W1) $\sqrt{1+x} = (1+x)^{\frac{1}{2}} =$

$$= 1 + \left(\frac{1}{2}\right)(x) + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\frac{(x)^2}{2!} + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\frac{x^3}{3!}$$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$$

(W2) $\sqrt{1-x} = (1-x)^{\frac{1}{2}}$

$$= 1 + \left(\frac{1}{2}\right)(-x) + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\frac{(-x)^2}{2!} + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\frac{(-x)^3}{3!}$$

$$= 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16}$$

$$\frac{\overset{W1}{\sqrt{1+x}} - \overset{W2}{\sqrt{1-x}}}{2x} = \frac{\left(1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}\right) - \left(1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16}\right)}{2x}$$

$$= \frac{\cancel{1} + \frac{x}{2} - \cancel{\frac{x^2}{8}} + \frac{x^3}{16} - \cancel{1} + \frac{x}{2} + \cancel{\frac{x^2}{8}} + \frac{x^3}{16}}{2x}$$

$$\frac{x + \frac{x^3}{8}}{2x}$$

$$\frac{x}{2x} + \frac{\frac{x^3}{8}}{2x}$$

$$\boxed{\frac{1}{2} + \frac{x^2}{16}}$$